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Curved branes and cosmological (a,b)-models

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Abstract: We construct p-brane solutions with non-trivial world volume metrics and show that applied to supergravity theories, they will lead to threshold BPS bound states of intersecting solutions. However applied to certain specific values of the couplings in cosmological (a,b)-models non-trivial solutions can be constructed.

Keywords: p-branes, D-branes.

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Contents

1. Introduction

It is very well known that supergravity theories have solutions in the form of extended object, the so-called p-branes [1]. They appear as solitonic objects that couple to the metric $\hat{g}_{\mu\nu}$, dilaton $\hat{\phi}$ and $(p+1)$-form gauge fields $\hat{C}$ of the theory [2]. In general they are of the following form:

$$\begin{aligned} d\hat{s}^2 &= H^{\alpha}\eta_{mn}dx^m dx^n - H^{\beta}\delta_{ij}dy^i dy^j \,, \\ e^{-2\hat{\phi}} &= H^{\gamma} \,, \\ \hat{C}_{01\ldots p} &= \delta H^{-1} \,, \end{aligned} \tag{1.1}$$

where the parameter α, β, γ and δ are determined by the type of p-brane and the specific supergravity theory considered. The coordinates x^m are the world volume coordinates and y^i the coordinates transverse to the brane. The above solution generally breaks half of the spacetime supersymmetry.

There have been several attempts to generalise the above solution by trying to write down more general metrics $\bar{h}_{ij}$ over the transverse space than the conformally flat metric shown above [3, 4]. It was demonstrated that this can be done as long as the transverse space is Ricci-flat and that part of the supersymmetry is preserved in the presence of Killing spinors. On the other hand, in [5] a solution was presented of the near-horizon limit of an M2-brane wrapped over an arbitrary genus surface.

Recently there has been a attempt [6] to generalize the standard $D8$-brane solution of [7] by imposing a general metric $\tilde{g}_{mn}$ on the world volume of the domain wall. Also here it was found that the equations of motion and supersymmetry were satisfied, provided that the metric $\tilde{g}_{mn}$ is Ricci-flat and allows covariantly constant Killing spinors.[1] It was shown that such solutions can be constructed, choosing as world volume a manifold with the appropriate holonomy groups.

[1] Shortly after this paper, another paper [8] on Ricci-flat branes appeared.

Here we want to take a different way and try to look for solutions to the Einstein-dilaton action coupled to gauge fields. We will show that for standard supergravity theories, i.e. in the absence of a cosmological constant, these solutions exist, but can be seen as solutions made out of intersecting branes. However, curved p-branes can be used to construct solutions to general Einstein–dilaton-gauge field theory with a cosmological constant, the so-called cosmological (a, b)-models, for specific values of the couplings a and b. These theories in general are not supersymmetric. Nevertheless, it is interesting to see how curved branes fit into this setup to provide a new class of solutions.

The organisation of this letter is as follows: in section 2 we will set our notation by giving a general ansatz for curved p-brane and compute the various curvature tensors of our ansatz. In section 3, we will look at the equations of motion of the curved p-branes in a general supergravity theory, reproduce the results of [3, 4, 6] and give solutions satisfying the conditions. In section 4 we will apply the ansatz to construct solutions to cosmological (a, b)-models.

2. The curved p-brane ansatz

We start with the following ansatz for the curved p-brane metric in D dimensions:

$$d\hat{s}^2 = H^{\alpha}(y)\tilde{g}_{mn}(x)dx^m dx^n - H^{\beta}(y)\bar{h}(y)_{ij}dy^i dy^j \,. \tag{2.1}$$

We allow the internal metric $\tilde{g}_{mn}(x^m)$ over the world volume to depend on the world volume coordinates x^m and the transverse space to have a more general geometry, described by the external metric $\bar{h}_{ij}(y)$. The parameters α and β are to be choosen such that in the case of flat internal and external metrics ($\tilde{g}_{mn} = \eta_{mn}$ and $\bar{h}_{ij} = \delta_{ij}$), the standard ("flat") p-brane solutions (1.1) are recovered. In that case the function $H(y)$ is harmonic in the transverse coordinates: $\partial_i\partial_i H = 0$.

It is straight forward to compute the Ricci tensor and the Ricci scalar for the metric (2.1):

$$\begin{aligned}
\hat{R}_{mn} &= \tilde{R}_{mn} - \frac{\alpha}{2}\tilde{g}_{mn}H^{\alpha-\beta-1}\Big[\bar{\nabla}_i\bar{\partial}^i H + (\tilde{D}-\beta-1)(\bar{\partial}H)^2\Big]\,,\\
\hat{R}_{im} &= 0\,,\\
\hat{R}_{ij} &= \bar{R}_{ij} + \frac{\beta}{2}H^{-1}\bar{\nabla}_k\bar{\partial}^k H\bar{h}_{ij} + (\tilde{D}-\beta)H^{-1}\bar{\nabla}_i\partial_j H + \\
&\quad + \frac{\beta}{2}(\tilde{D}-\beta-1)H^{-2}(\bar{\partial}H)^2\bar{h}_{ij} + \\
&\quad + \left[\frac{\alpha^2}{4}(p+1) + \frac{\beta^2}{4}(D-p-1) - \tilde{D}(\beta+1) + \frac{\beta^2}{2} + \beta\right]H^{-2}\partial_i H\partial_j H\,,\\
\hat{R} &= H^{-\alpha}\tilde{R} - H^{-\beta}\bar{R} - (2\tilde{D}-\beta)H^{-\beta-1}\bar{\nabla}_i\bar{\partial}^i H - \\
&\quad - \left[\frac{\alpha^2}{4}(p+1) + \frac{\beta^2}{4}(D-p-1) + \tilde{D}(\tilde{D}-2\beta-2) + \frac{\beta^2}{2} + \beta\right]H^{-\beta-2}(\bar{\partial}H)^2\,.
\end{aligned} \tag{2.2}$$

Here $\tilde{R}_{mn}(\bar{R}_{ij})$ and $\tilde{R}(\bar{R})$ are the Ricci tensor and the Ricci scalar of the internal (external) metric, p is the number of spatial dimensions of the brane and

$$\tilde{D} = \frac{\alpha}{2}(p+1) + \frac{\beta}{2}(D-p-1)\,. \tag{2.3}$$

We denote by $\bar{\nabla}_i$ the covariant derivative with respect to the metric $\bar{h}_{ij}$. Furthermore we have defined

$$\bar{\partial}^i H = \bar{h}^{ij}\partial_j H\,, \qquad (\bar{\partial}H)^2 = \bar{h}^{ij}\partial_i H \partial_j H\,. \tag{2.4}$$

Note that the overall Ricci tensor and Ricci scalar (2.2) factorize into parts coming from the internal and external metric and a part coming from the overall metric of the brane.

3. Ricci-flat branes and intersections

We will now look at the concrete example of a p-brane in supergravity. Our ansatz for the curved p-brane solution is given by:

$$\begin{aligned} d\hat{s}^2 &= H^\alpha[\tilde{g}_{mn}dx^m dx^n] - H^\beta[\bar{h}_{ij}dy^i dy^j]\,, \\ e^{-2\hat{\phi}} &= H^\gamma\,, \\ \hat{C}_{01\ldots p} &= \sqrt{|\tilde{g}|}H^{-1}\,. \end{aligned} \tag{3.1}$$

Note that the gauge fields are all given in the electric formulation, where the dual potentials are used for magneticly charged p-branes ($p > 3$). The extra factor $\sqrt{|\tilde{g}|}$ has been added to compensate for the curvature of the internal metric in the equations of motion. The equation of motion of $\hat{C}$ is satisfied if the function $H(y)$ is harmonic on the external metric $\bar{h}_{ij}$:

$$\bar{\nabla}_i \bar{\partial}^i H = 0\,. \tag{3.2}$$

The Einstein and dilaton equations factorize completely into a part coming from the internal and external metrics and a part that is identical zero, being the equations of motion of the "flat" p-brane, upto eq. (3.2). Therefore we have that:

$$\begin{aligned} \tilde{R}_{mn} &= 0\,, \\ \bar{R}_{ij} &= 0\,, \\ H^{-\alpha}\tilde{R} - H^{-\beta}\bar{R} &= 0\,. \end{aligned} \tag{3.3}$$

These are the Ricci-flatness conditions given in [3, 4, 6]. Also the supersymmetry variations factorize in a part proportional to the projection operator of the p-brane and the following conditions on ϵ coming from the internal and external metrics:

$$\tilde{\nabla}_m \epsilon = 0\,, \qquad \bar{\nabla}_i \epsilon = 0\,. \tag{3.4}$$

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The problem now consists of finding (non-trivial) $(p+1)$- or $(D-p-1)$-dimensional manifolds admitting Killing spinors. These are given in terms of their holonomy groups: only those manifolds are allowed that have holonomy groups Spin(7), G_2, SU(3) and Sp(1) (or subgroups thereof), depending on the dimension of the manifold considered [6, 8].

Here however we want to take a different way and try to satisfy the conditions (3.3)–(3.4) by looking at Ricci-flat supersymmetric solutions of the $(p+1)$- or $(D-p-1)$-dimensional Einstein-dilaton-gauge field lagrangean. To our knowlegde, there are two such solutions: the Kaluza-Klein monopole [9] and the gravitational wave [10].

Choosing for the internal metric $\tilde{g}_{mn}$ the four-dimensional gravitational wave, the generalized p-brane (3.1) takes the form:

$$\begin{aligned} d\hat{s}^2 &= H^\alpha[(2-F)dt^2 - Fdz^2 - 2(1-F)dtdz - dx_a^2] - H^\beta d\vec{y}^2\,, \\ e^{-2\hat{\phi}} &= H^\gamma\,, \\ \hat{C}_{01\ldots p} &= H^{-1}\,. \end{aligned} \tag{3.5}$$

This solution satisfies the equations of motion if the functions F and H are harmonic in x^a and y^i respectively and preserves one quarter of supersymmetry. However it is not difficult to see that this is actually the solution of a theshold BPS intersection of a p-brane and a ten-dimensional wave in its world volume [11, 12] (in the notation of [13]):

$$(1|p,\mathcal{W}) = \left\{ \begin{array}{c|cccccccccc} \times & \times & \times & \times & \times & \times & - & - & - & - & :H \\ \times & z & - & - & - & - & - & - & - & - & - \;:F \end{array}\right. \qquad \underbrace{}_{t,z}\;\underbrace{}_{x_a}\;\underbrace{}_{y_i} \tag{3.6}$$

Also the inclusion of a Kaluza-Klein monopole in the transverse space will lead to a known threshold BPS intersection of a p-brane and a monopole [11, 12, 13]

$$(p|p,\mathcal{KK}) = \left\{ \begin{array}{c|ccccccccc} \times & \times & \times & \times & - & - & - & - & - & - \\ \times & \times & \times & \times & \times & \times & z & A_1 & A_2 & A_3 \end{array}\right. . \tag{3.7}$$

The dependence of the harmonic functions H and F (in the language of intersecting branes, on the overall transverse and relative transverse coordinates respectively, i.e. the gravitational wave delocalised in its overall transverse directions) is the correct one to belong to the class of threshold BPS bound states [14].

A logical next step would be trying to weaken the Ricci-flatness condition by exciting other fields with dependences on the world volume coordinates (For example excite a U(1) gauge field and construct a extreme Reissner-Nördström solution on the world volume). However, these generalisations turn out to be impossible: adding

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extra gauge fields or dilaton with dependence on the world volume coordinates inevitably leads to terms in the equations of motion which cannot be cancelled. For example adding a dilaton $\hat{\phi} \sim \tilde{\chi}(x)$ will lead in the Einstein equations to a term

$$\hat{\nabla}_i \partial_m \hat{\phi} = \hat{\Gamma}^n_{im} \partial_n \tilde{\chi} = \frac{1}{4} H^{-1} \partial_i H \partial_m \tilde{\chi} \,, \tag{3.8}$$

which clearly cannot be cancelled by other terms. It would be interesting to see whether the Ricci-flatness condition can be circumvented by coupling a curved p-brane to a world volume action and let the Born-Infeld field on the D-brane play the role of U(1) gauge field of an extreme Reissner-Nördström solution.

4. Curved branes and cosmological (a, b)-models

As was pointed out in the previous section, the Ricci-flatness condition is restriction, leading either to the class of branes with world volumes manifolds with the adequate holonomy groups, or to threshold BPS bound state intesections of p-branes with gravitational waves or monopoles.

However, if one forgets for one moment the Ricci-flatness condition imposed by the first two equations of (3.3), the dilaton equation admits a more general solution:

$$\tilde{R} = \kappa \,, \qquad \bar{R} = \kappa H^{\beta - \alpha} \,, \tag{4.1}$$

where κ is an arbitrary constant. The obvious way now to make this solution also fit the Einstein equations of (3.3), is to modify these equations by introducing a cosmological constant in the theory.

So we can use curved branes to obtain solutions of theories with cosmological constants. The idea consists of taking a p-brane solution of a theory with zero cosmological constant, impose on the world volume of the brane a metric with (non-zero) constant curvature and see whether the new ansatz satisfies the equations of motion of the theory with non-zero cosmological constant. In other words, the question is whether we can relate the cosmological constant of the world volume metric to the cosmological constant of the full theory.

The most general theory in D dimensions of gravity coupled to a dilaton and a $(p+1)$-form gauge field in the presence of a cosmological constant is the so-called cosmological (a, b)-model:

$$\mathcal{L}_D = \sqrt{|\hat{g}|} \left\{ e^{-2\hat{\phi}} \left[\hat{R} - 4(\partial \hat{\phi})^2 \right] + \frac{(-)^{p+1}}{2(p+2)!} e^{a\hat{\phi}} \hat{F}^2_{(p+2)} + e^{b\hat{\phi}} \Lambda \right\} . \tag{4.2}$$

The parameters a, b are constants parametrising the dilaton potential that couples to the gauge fields and the cosmological constant. Note that the above lagrangean will only be supersymmetric for specific values of a and b. For example, for $D = 10$, $a = b = 0$, we recover Romans' massive supergravity theory [15], and for $D < 10$, $a = b = -2$ massive heterotic supergravity [16, 17].

The equations of motion for this model are given by:

$$\begin{aligned}
\hat{R}_{\mu\nu} - \frac{b+2}{4} e^{(b+2)\hat{\phi}} \Lambda \hat{g}_{\mu\nu} - 2\hat{\nabla}_\mu \partial_\nu \hat{\phi} - \frac{1}{(p+1)!} e^{(a+2)\hat{\phi}} \hat{T}_{\mu\nu}(\hat{F}) &= 0\,, \\
\hat{R} + 4(\partial\hat{\phi})^2 - 4\hat{\nabla}^2\hat{\phi} - \frac{a}{4(p+1)!} e^{(a+2)\phi} \hat{F}^2 - \frac{b}{2} e^{(b+2)\phi}\Lambda &= 0\,, \\
\hat{\nabla}_\mu [e^{a\phi} \hat{F}^{\mu\rho_1\ldots\rho_{p+1}}] &= 0\,,
\end{aligned} \tag{4.3}$$

where $\hat{T}_{\mu\nu}(\hat{F})$ is the energy-momentum tensor of the gauge field

$$\hat{T}_{\mu\nu}(\hat{F}) = \hat{F}_{\mu\rho_1\ldots\rho_{p+1}} \hat{F}_\nu{}^{\rho_1\ldots\rho_{p+1}} - \frac{1}{2(p+2)} \hat{g}_{\mu\nu} \hat{F}^2\,. \tag{4.4}$$

A general p-brane solution in $D > 2$ for the case of $\Lambda = 0$ is of the form [18, 19]:

$$\begin{aligned}
d\hat{s}^2 &= H^\alpha dx_m^2 - H^\beta dy_i^2\,, \\
e^{-2\hat{\phi}} &= H^\gamma\,, \\
\hat{F}_{01\ldots pi} &= \delta \partial_i H^{-1}\,,
\end{aligned} \tag{4.5}$$

with the parameters α, β, γ and δ taking the values

$$\begin{aligned}
\alpha &= \frac{2-a}{N}\,, \qquad \beta = -\frac{2+a}{N}\,, \qquad \delta^2 = -\frac{4}{N}\,, \\
\gamma &= -\frac{1}{N}\left[2(p+1) - \frac{1}{2}(a+2)(D-2)\right]\,, \\
N &= (p+1)a - \frac{1}{2}(D-2)\left(1 + \frac{a}{2}\right)^2\,.
\end{aligned} \tag{4.6}$$

The aim is now to put a non-trivial world volume metric on these brane solutions and see whether we can construct in this way solutions to the full equations of motion (4.3). These solutions are of course not the most general solutions to these equations. On the contrary, we are looking at a very specific class of solutions to cosmological (a, b)-models, involving curved p-branes. Other p-brane solutions to various cosmological theories where given in [20, 21, 22, 17, 23].

Plugging ansatz (3.1) into the equations (4.3), we obtain, upto the equations of motion of the "flat" p-brane:

$$\begin{aligned}
\hat{R} - \frac{b}{2} e^{(b+2)\hat{\phi}} \Lambda &= H^{-\alpha} \tilde{R} - H^{-\beta} \bar{R} - \frac{b}{2} H^{-\frac{b+2}{2}\gamma} \Lambda \equiv 0\,, \\
\hat{R}_{mn} - \frac{b+2}{4} e^{(b+2)\hat{\phi}} \Lambda \hat{g}_{mn} &= \tilde{R}_{mn} - \frac{b+2}{4} \Lambda H^{\alpha - \frac{b+2}{2}\gamma} \tilde{g}_{mn} \equiv 0\,, \\
\hat{R}_{ij} - \frac{b+2}{4} e^{(b+2)\hat{\phi}} \Lambda \hat{g}_{ij} &= \bar{R}_{ij} + \frac{b+2}{4} \Lambda H^{\beta - \frac{b+2}{2}\gamma} \bar{h}_{ij} \equiv 0\,.
\end{aligned} \tag{4.7}$$

These equations can be satisfied if

$$\tilde{R}_{mn} = -\frac{1}{D-2}\Lambda\tilde{g}_{mn}\,, \qquad \bar{R}_{ij} = \frac{1}{D-2}\Lambda H^{\beta+\frac{2\gamma}{D-2}}\bar{h}_{ij}\,,$$
$$\gamma = -\frac{D-2}{2}\alpha\,, \qquad b = -\frac{2D}{D-2}\,, \tag{4.8}$$

Note that the parameter b only depends on the dimension of the overall space: for every dimension there is a well-defined cosmological dilaton potential for which curved cosmological p-branes are allowed. The condition for $\tilde{g}_{mn}$ is the expression for the Ricci tensor of a constant curvature space, for example a $(p+1)$-dimensional (anti)-de Sitter space, with cosmological constant $\frac{\Lambda}{D-2}$ (depending on the sign of Λ). The condition for the metric $\bar{h}_{ij}$, together with the harmonicity condition (3.2), is much more involved and it is not clear what the solution is to these equations.

Combining the conditions (4.6) for the "flat" p-brane solution with the above conditions (4.8), we find that we have cosmological solutions for curved p-branes with

$$p = D-3\,. \tag{4.9}$$

Let us now look at some explicite examples:

- **Romans' theory:** It is easy to see that there are no solutions of the above class in ten-dimensional massive type-IIA supergravity, since the condition (4.8) on b is not compatible with the coupling of the cosmological constant in Romans' theory ($b=0$). Note that the curved D8-brane solutions of [6] is not found in this setup. This is because the curved D8-brane falls outside our ansatz: the "flat" D8-brane is not a solution of the type given in (4.5)–(4.6).

- **Massive heterotic supergravity:** From eq. (4.8) it follows that b can not take the value -2 in this setup. Therefore there are no curved cosmological p-branes in massive heterotic supergravity.

- **p-branes in $D=10$:** From eq. (4.9), we see that the type-IIB D7-brane coupled to RR 9-form ($a=0$) can be embedded in a cosmological theory, with dilaton coupling for the cosmological term $b=-5/2$, the only coupling allowed in ten dimensions in this setup. Of course, this cosmological theory is no longer a supergravity theory. If we choose for the constant curvature surface on the world volume an eight-dimensional AdS space in horospheric coordinates, the curved cosmological D7-brane is of the form

$$d\hat{s}^2 = H^{-1/2}\left[\frac{56}{\Lambda x_7^2}(dt^2 - dx_1^2 - \cdots - dx_7^2)\right] - H^{1/2}\bar{h}_{ij}dy^i dy^i\,,$$
$$e^{-2\hat{\phi}} = H^2\,,$$
$$\hat{F}_{01\ldots 7i} = \partial_i H^{-1}\,, \tag{4.10}$$

where the metric $\bar{h}_{ij}$ has to satisfy $\bar{R}_{ij} = \frac{\Lambda}{8}H\bar{h}_{ij}$.

- p**-branes in** $D = 6$**:** In six dimensions, we can embedd the curved D3-brane in a cosmological theory with dilaton coupling $b = -3$:

$$\begin{aligned} d\hat{s}^2 &= H^{-1}\left[\frac{12}{\Lambda x_3^2}(dt^2 - dx_1^2 - \cdots - dx_3^2)\right] - H\bar{h}_{ij}dy^i dy^i \,, \\ e^{-2\hat{\phi}} &= H^2 \,, \\ \hat{F}_{01\ldots 7i} &= \sqrt{2}\partial_i H^{-1} \,. \end{aligned} \tag{4.11}$$

- p**-branes in** $D = 4$**:** In four dimensions, strings can be embedded in a theory with dilaton coupling $b = -4$.

- $\beta = -\frac{2\gamma}{D-2}$: For this value condition (4.8) on $\bar{R}_{ij}$ would become very simple. However, as can be seen from eqns. (4.6), the constraint $\alpha = \beta$ leads to inconsistencies.

5. Conclusions

We showed that putting a Ricci-flat metric in the world volume of a p-brane in supergravity leads to BPS threshold intersections of the considered p-brane with a gravitational wave or a Kaluza-Klein monopole. Going beyond the Ricci-flatness condition we found a new class of curved p-branes solutions in theories with a particular coupling of the cosmological constant.

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